Symmetry algebras of normal $\mathcal{A}$-hypergeometric systems

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Abstract. The structure of the symmetry algebras of normal $\mathcal{A}$-hypergeometric systems is studied and determined in terms of generators and relations. An irreducible component of the semisimple part of their symmetry Lie algebras is proved to be either of A-type or of C-type. This result generalizes Hrabowski's theorem [Hr].

Key words: $\mathcal{A}$-hypergeometric system, symmetry algebra, symmetry Lie algebra.

Introduction

Miller demonstrated in [M1] that a variety of addition theorems and generating functions for generalized hypergeometric functions were derived from the representation theory of the corresponding symmetry Lie algebras, which he called dynamical symmetry algebras. In the case of A_n-type, by using the symmetry Lie algebras of hypergeometric systems, Sasaki obtained in [Sas] all contiguity relations for the corresponding generalized hypergeometric functions, and Horikawa clarified in [Hor] the action of the Weyl group on the space of those functions. These examples show that the study of the structure and the representation theory of the symmetry Lie algebras of hypergeometric systems is very important. In this direction, Hrabowski proved in [Hr] that, when a symmetry Lie algebra generates all symmetries, it is a simple Lie algebra of finite dimension if and only if the simple Lie algebra is either of A_n-type or of C_n-type where n is the dimension of the parameter space. In this paper, we generalize his result when a symmetry Lie algebra not necessarily generates all symmetries. To solve this problem, we proceed in the following way. First we determine the structure of the associative algebra composed of all symmetries; we call this associative algebra, the symmetry algebra of a hypergeometric system. Next we study the symmetry Lie algebra as the Lie subalgebra composed of all symmetries of order less than or equal to one.

Among a number of definitions of generalized hypergeometric systems, we choose a definition suitable for our problem of determining the structure