The space $N(\sigma)$ and the F. and M. Riesz theorem
(Dedicated to Professor Satoru Igari on his sixtieth birthday)

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Abstract. We give a property of spectrum of measures on a certain LCA group. We also give a characterization of the space $N(\sigma)$ of measures on a LCA group under our setting.

Key words: locally compact abelian group, transformation group, measure, quasi-invariant measure, Fourier transform, spectrum.

1. Introduction

Let $(G, X)$ be a (topological) transformation group, in which $G$ is a locally compact abelian (LCA) group and $X$ is a locally compact Hausdorff space. Let $M(X)$ be the Banach space of bounded regular measures on $X$. Let $L^1(G)$ and $M(G)$ be the group algebra and the measure algebra respectively. $m_G$ stands for the Haar measure of $G$. $M_\sigma(G)$ denotes the subspace of $M(G)$ consisting of singular measures. Let $\sigma$ be a quasi-invariant, (positive) Radon measure on $X$, and let $N(\sigma) = \{\mu \in M(X) : h * \mu << \sigma \text{ for all } h \in L^1(G)\}$. For $\mu \in M(X)$, let $\text{sp}(\mu)$ be the spectrum of $\mu$. Let $\mu = \mu_a + \mu_s$ be the Lebesgue decomposition of $\mu$ with respect to $\sigma$.

We define two families $C_0 (= C_0(\sigma))$ and $C_0^0 (= C_0^0(\sigma))$ of closed sets $\hat{G}$ as follows:

$$C_0 = \{E \subset \hat{G} : \text{closed set, } \mu \in M(X), \text{sp}(\mu) \subset E \implies \text{sp}(\mu_s) \subset E\};$$

$$C_0^0 = \{E \in C_0 : \forall E' \subset E : \text{closed set} \implies E' \in C_0\}.$$

When $G$ is a compact abelian group, the notion of $C_0$ and $C_0^0$ is introduced in [5]. Finet and Tardivel-Nachef ([2]) obtained the following two results in case $G$ is a compact abelian group.

Proposition 1.1 (cf. [2, Proposition 4.9]). Suppose $G$ is a compact abe-