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Differential field extensions with no movable algebraic branches

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Abstract. Differential field extensions with no movable algebraic branches are defined, and such a differential field extension is proved to be a Painlevé-Umemura extension provided that it is included in a decomposable extension, which was defined previously by the author.

Key words: differential field, differential field extension, formal Laurent series.

1. Introduction

Let K be an ordinary differential field of characteristic 0 with the differentiation D. Let U be a universal extension of K. Any differential field extension of K under consideration is tacitly assumed to be finitely generated and embedded in U unless particularly mentioned.

For differential field extensions of finite type, namely, finitely generated in the sense of field extensions, the notion of decomposability is defined inductively. Finite extensions are decomposable. A differential field extension R/K with finite transcendence degree is decomposable if there exist a differential field extension L/K and an intermediate differential field M between LR and L with tr.deg. $M/L=1$ such that R and L are free over K and LR/M is decomposable (cf. [5]).

For differential field extensions of finite type, the notion of Painlevé-Umemura extensions, or briefly, PU-extensions is defined inductively. Finite extensions are PU. A differential field extension R/K is PU if there exist a differential field extension L/K and a constant c of LR transcendental over L such that R and L are free over K and $LR/L(c)$ is PU (cf. [7]).

Our objective is to show some type of differential subfield of a decomposable differential field extension of K turns out to be a PU-extension. Such an attempt was done in [9] so as to afford the second proof of the irreducibility for Painlevé's first transcendent. Related topics will be seen abundantly in [8]. To do that we need some notions.

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