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Besov spaces on symmetric manifolds

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Abstract. We investigate the spaces of Besov type on symmetric manifolds of the noncompact type. The paper is focused on finding the equivalent norms via the means typical for harmonic analysis on these manifolds.

Key words: Besov spaces, Helgason transform, symmetric spaces.

The whole scale of Besov spaces $B_{p,q}^{s}(X)$ on a Riemannian manifold with bounded geometry was defined by H. Triebel in 1986. The definition is of local nature. But in the case of symmetric manifolds of the non-compact type one can use the Fourier analysis similar as in the Euclidean case. In the paper we investigate the connection between the Besov spaces and the Helgason-Fourier transform on symmetric manifolds of the non-compact type for $p=2$. We focus on the problem of equivalent norms.

1. Preliminaries

1.1. Symmetric manifolds of the non-compact type

Let $X=G/K$ be a Riemannian symmetric manifolds of the noncompact type, i.e. G is a connected semi-simple Lie group with finite center and K is a maximal compact subgroup of G. We list briefly the customary notation associated with X and refer for example to [10] for more explicit definitions. Let $\mathfrak{g}$ and $\mathfrak{k}$ denote Lie algebras of G and K respectively. Their complexifications will be denoted by the subscript $\mathbf{C}$. Let $\mathfrak{p}$ be an orthogonal complement of $\mathfrak{k}$ in $\mathfrak{g}$ with respect to the Killing form $<,>$. Then

$$\mathfrak{g}=\mathfrak{k}\oplus\mathfrak{p} \quad \text{(the Cartan decomposition).} \tag{1}$$

We assume that the Riemannian metric on X is generated by $<,>$.

Let $\mathfrak{a}$ be a maximal abelian subspace of $\mathfrak{p}$ and $\mathfrak{a}^{\star}$ its dual. An element $\lambda\in\mathfrak{a}^{\star}$ is called a restricted root of $\mathfrak{g}$ if $\lambda\neq 0$ and the corresponding root space $\mathfrak{g}_{\lambda}=\{X\in\mathfrak{g}:[H,X]=\lambda(H)X, \text{ for all } H\in\mathfrak{a}\}$ is not trivial. The

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