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On a super class of p-hyponormal operators

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Abstract. Given an operator A on a Hilbert space $\mathcal{H}, A$ is said to be p-hyponormal $(0<p\leq 1)$ if $(AA^{\star})^{p}\leq(A^{\star}A)^{p}$. The class $H(p)$ of p-hyponormal operators has been studied in a number of papers in the recent past. Let $K(p)$ denote the class of operators A for which $((AA^{\star})^{p}x, x)\leq\Vert x\Vert^{2(1-p)}\ (A^{\star}Ax, x)^{p}$ for all $x\in \mathcal{H}$. Then $H(p)\subset K(p)$. In this note we study the spectral properties of operators in $K(p)$, and show that a number of the properties enjoyed by hyponormal operators carry over to $K(p)$. Our arguments often lead to an alternative, sometimes simpler, proof of the results for $H(p)$.

Key words: p-hyponormal operators, class $K(p)$, spectral properties.

1. Introduction

We consider operators (i.e., bounded linear transformations) on a complex Hilbert space $\mathcal{H}$. The operator A is said to be p-hyponormal, $0<p\leq 1$, if $(AA^{\star})^{p}\leq(A^{\star}A)^{p}$. It is an easy consequence of the Löwner inequality that a p-hyponormal operator is q-hyponormal for all $0<q\leq p$. In particular, a 1-hyponormal (or simply hyponormal) operator is p-hyponormal for all $0<p<1$, and in studying p-hyponormal operators for a general $0<p<1$ it is sufficient to consider $0<p\leq\frac{1}{2}$. Semi-hyponormal (or, $\frac{1}{2}$-hyponormal) operators were introduced by Xia [20], and p-hyponormal operators for $0<p<\frac{1}{2}$ were first studied by Aluthge [1]. Recently there have been a number of papers, especially by Muneo Cho et al. [2, 3, 4, 5, 6] and Masatoshi Fujii et al. [10, 11], on p-hyponormal operators, their spectral properties and their relationship to other classes of operators. Generally speaking p-hyponormal have properties very similar to hyponormal operators [1, 2, 3, 4, 5, 6, 10, 20, 21].

Let $H(p)$ denote the class of p-hyponormal operators, $0<p\leq\frac{1}{2}$. Then $((AA^{\star})^{p}x, x)\leq((A^{\star}A)^{p}x, x)\leq\Vert x\Vert^{2(1-p)}(A^{\star}Ax, x)^{p}$ for all $x\in \mathcal{H}$. Let $K(p)$, $0<p\leq\frac{1}{2}$, denote the class of operators A for which $((AA^{\star})^{p}x, x)\leq \Vert x\Vert^{2(1-p)}\ (A^{\star}Ax, x)^{p}$. Then $H(p)\subset K(p)$ and the class $K(p)$ is monotone decreasing on p; also, operators $A\in K(p)$ are paranormal, i.e., if $A\in$

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