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The Schrödinger operator with random vector potentials

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Abstract. We consider a non-Gaussian probability measure on $(\mathcal{S}'(\mathbf{R}^d; \mathbf{R}^d), \mathcal{B}(\mathcal{S}'(\mathbf{R}^d; \mathbf{R}^d))$ whose characteristic functional is given by Lévy-Khinchine formula. We construct a one parameter semi-group whose generator is expressed by "$(\partial - iA)^2$", $A \in \mathcal{S}'(\mathbf{R}^d; \mathbf{R}^d)$ formally. It is also shown that the generator is self-adjoint.

Key words: electromagnetic fields, semi-group process, gauge covariance.

1. Introduction

Nonlinear electromagnetism is a theory of generalized random fields in four dimensional Euclidean space-time obtained by solving a system of coupled stochastic partial differential equations [AHK1, 2, AHKI, AIK1]. The fields are homogeneous with respect to the Euclidean group. It has been shown in [I, AIK3] that the fields have the sharp global Markov property. In [AIK2], relativistic time ordered functions are constructed in the model. The theory includes the usual theory of electromagnetic fields as a special case. As other special cases, it describes a class of models where the fields confine charges in Wilson's sense, and it has mild ultraviolet behavior [Ta]. Recently, in [AT], they consider a coupled theory of the confining nonlinear electromagnetic field and a charged scalar field within the quenched approximation. And they consider the propagator of matter field in approximation. The propagator is the expectation value of the resolvent kernel of the Schrödinger operator with the vector potential of the electromagnetic field. They define it as an element of $L^p(\mathcal{S}'(\mathbf{R}^d; \mathbf{R}^d), \mu)$ where μ is the probability measure which characterizes the nonlinear electromagnetic field. And they examined the asymptotic behavior of the propagator and showed that the correlation length is zero. However, they did not define the operator.

In this paper we consider the semi-group with random vector potential formally represented by $e^{t/2(\partial - iA)^2}$ on $L^2(\mathbf{R}^d)$, $d \geq 3$, $A \in \mathcal{S}'(\mathbf{R}^d; \mathbf{R}^d)$. The word "formally" reflects the fact we have not given a simultaneous definition of the operator $e^{t/2(\partial - iA)^2}$ for every A in the support of the measure μ on

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