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Regularly varying correlation functions and KMO-Langevin equations

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Abstract. We study a variant of Okabe's first KMO-Langevin equation. After establishing unique existence of a stationary solution, we precisely describe the long-time behavior of the correlation function R of the solution. In particular, the behavior such as $R(t) \sim ct^{-1}$ as $t \to \infty$ is characterized by using Π-variation. Correlation functions regularly varying with index $p \in [-1, 0)$ are characterized in terms of outer functions.

Key words: first KMO-Langevin equation, stationary process, reflection positivity, correlation function, outer function, regular variation, Π-variation, stationary random distribution.

1. Introduction

In [O4], Okabe introduced the linear stochastic delay equation

$$\dot{X}(t) = -\beta X(t) - \int_{-\infty}^{t} \gamma(t-s)\dot{X}(s)ds + \alpha\dot{B}(t). \tag{1.1}$$

This equation is called a *first KMO-Langevin equation*. Here, α and β are positive numbers, $\dot{B}$ is a Gaussian white noise, and the kernel function $\gamma : (0, \infty) \to [0, \infty)$ has a representation of the form

$$\gamma(t) = \int_0^\infty e^{-t\lambda} d\rho(\lambda) \qquad (t > 0), \tag{1.2}$$

where ρ is a Borel measure on $(0, \infty)$ such that

$$\int_0^\infty \frac{1}{\lambda + 1} d\rho(\lambda) < \infty. \tag{1.3}$$

The key feature of equation (1.1) is that it describes the time evolution of a stationary Gaussian process X with *reflection positivity*: the correlation function R of X, which is defined by $R(t) := E[X(t)X(0)]$, takes the form

$$R(t) = \int_0^\infty e^{-|t|\lambda} d\sigma(\lambda) \quad (t \in \mathbb{R}), \tag{1.4}$$

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