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Generating alternating groups

Naoki CHIGIRA

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Abstract. We will give an elementary proof of the following: For any nonidentity element x in the alternating group A_n on n symbols, there exists an element y such that x and y generate A_n.

Key words: the alternating group, block.

Let S_n be the symmetric group on the symbols $\Omega=\{1,2,\ldots,n\}$ and A_n the alternating group on Ω. Isaacs and Zieschang [1] give an elementary proof of the following:

Theorem A *Assume that $n\neq 4$ and let $x\in S_n$ be an arbitrary nonidentity element. Then there exists an element $y\in S_n$ such that $S_n=\langle x, y\rangle$.*

They say "*A result similar to Theorem* A *is known to be valid for the alternating group A_n for all values of n. Although it seems likely that a proof of this result along the lines of our proof of Theorem* A *might exist, there are technical difficulties in some cases, and we have not actually found such a proof.*"

In this note, we will give a proof for A_n along the lines of the proof of Theorem A by Isaacs and Zieschang [1].

Theorem *Let $x\in A_n$ be an arbitrary nonidentity element. Then there exists an element $y\in A_n$ such that $A_n=\langle x, y\rangle$.*

A nonempty subset $\Delta\subseteq\Omega$ is said to be a block for G if Δ^x is either disjoint from or equal to Δ for each element $x\in G$. A group G is said to be primitive if the only blocks for G are the singleton subset or the whole set Ω.

The following theorems and lemma play an important role in our proof.

Theorem (Jordan) *Suppose that G is a primitive subgroup of S_n. If G contains a 3-cycle, then either $G=S_n$ or $G=A_n$.*

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