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An oscillation result for a certain linear differential equation of second order

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Abstract. We consider the second order equation $f'' + (e^{P_1(z)} + e^{P_2(z)} + Q(z))f = 0$, where $P_1(z) = \zeta_1 z^n + \dots$, $P_2(z) = \zeta_2 z^n + \dots$, are non-constant polynomials, $Q(z)$ is an entire function and the order of Q is less than n. Bank, Laine and Langley studied the cases when $Q(z)$ is a polynomial and ξ_2/ξ_1 is either non-real or real negative, while the author and Tohge studied the cases when $\xi_1 = \xi_2$ or ξ_2/ξ_1 is non-real. In this paper we treat the case when ζ_2/ζ_1 is real and positive.

Key words: complex oscillation theory, Nevanlinna theory, Value distribution.

1. Introduction

We are concerned with the zero distribution of solutions of some linear differential equations of second order

$$f'' + A(z)f = 0, \tag{1.1}$$

where $A(z)$ is an entire function. We assume that the reader is familiar with the standard notation in Nevanlinna theory (see e.g. [8], [10], [11]). Let f be a meromorphic function. As usual, $m(r, f)$, $N(r, f)$, and $T(r, f)$ denote the proximity function, the counting function, and the characteristic function of f, respectively. We denote by $S(r, f)$ any quantity of growth $o(T(r, f))$ as $r \to \infty$ outside of a possible exceptional set of finite linear measure. We use the symbols $\sigma(f)$ to denote the order of f, and $\lambda(f)$ to denote the exponent of convergence of the zero-sequence of f. The studies and problems on complex oscillation theory are found in, for instance, Laine [10, Chapter 3–8] and Yang, Wen, Li and Chiang [14, pp. 357–358].

This note is devoted to the study of the equation (1.1) in the case

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