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Extrinsic shape of circles and the standard imbedding of a Cayley projective plane

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Abstract. The main purpose of this paper is to give a characterization of the parallel imbedding of a Cayley projective plane $P_{Cay}(c)$ into a real space form in terms of the extrinsic shape of particular circles on $P_{Cay}(c)$.

Key words: cayley projective plane, parallel imbedding, cayley circle, totally real circle.

1. Introduction

To what extent can we determine the properties of a submanifold by observing the extrinsic shape of geodesics or circles of a submanifold? As typical cases, we recall that a submanifold is totally geodesic (resp. totally umbilic with parallel mean curvature vector) if and only if *all* geodesics (resp. circles) of the submanifold are geodesics (resp. circles) in the ambient space ([7]).

On the other hand, it is well-known that a sphere is the only surface in E^3 all of whose geodesics are circles in E^3. This result is generalized as follows: A submanifold of a real space form is isotropic and parallel if and only if all geodesics of the submanifold are circles in the ambient space ([4], [9]).

Then, what is the extrinsic shape of circles of an isotropic parallel submanifold of a real space form? An isotropic parallel submanifold of a real space form is locally equivalent either to the first standard imbedding of one of the compact symmetric spaces of rank one or to the second standard imbedding of a sphere. It is proved in [3] that the image of a circle under the first standard imbedding of a real projective space or the second standard imbedding of a sphere is *never* a circle in the ambient space. On the contrary, *some* circles of a complex projective space or a quaternionic projective space are mapped to circles in the ambient space under the first standard imbedding ([1]).

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