Hokkaido Mathematical Journal Vol. 26 (1997) p. 405–409

On defect groups of the Mackey algebras for finite groups

Fumihito ODA

(Received June 2, 1996)

Abstract. In this paper, we introduce a new Mackey functor $\mathcal{T}$ and give a relation of ordinary defect group and defect group of the Mackey algebra of a finite group.

Key words: Mackey algebra, Mackey functor, group representation, block, defect group.

1. Introduction

The Mackey algebra $\mu_R(G)$ of a finite group G over a commutative ring R introduced by J. Thévenaz and P.J. Webb [TW] for studying the structure of Mackey functors. This is an algebra of finite rank over R with the property that the category of Mackey functors of G over R is equivalent to the category of left $\mu_R(G)$-modules. So Thévenaz and Webb studied the blocks of Mackey functors in terms of the simple Mackey functors. In [TW] they determined the division of the simple Mackey functors into blocks of Mackey functors.

On the other hand, Yoshida introduced the span ring of the category of finite G-sets and gave the formula of the centrally primitive idempotents of the span ring [Yo]. It is interesting that the Mackey algebra $\mu_R(G)$ is isomorphic to the span ring of the category of finite G-sets. We can apply the formula of the span ring to the Mackey algebra $\mu_R(G)$. A centrally primitive idempotent of the span ring is indexed by the p-perfect subgroup J and the p-block of $N_G(J)/J$. In particular, we consider that the p-blocks of the group algebra of G is the corresponding centrally primitive idempotents of the span ring indexed by the trivial subgroup and p-blocks of $N_G(1)/1 = G$.

In this paper, we consider a defect group of the blocks of Mackey functors of G like as the ordinary block theory. The word "blocks of Mackey functors" means two-sided direct summands of $\mu_R(G)$ or the corresponding centrally primitive idempotents of $\mu_R(G)$. We introduce a Mackey functor $\mathcal{T}$ for the sake of the definition of a defect groups of blocks of Mackey functors. The inductions of $\mathcal{T}$ are generalization of the trace maps of the group

1991 Mathematics Subject Classification : Primary 20C20; Secondary 19A22, 20J05, 16G20.