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Seminormal composition operators induced by affine transformations

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Abstract. A class of composition operators on $L^2(\mu)$-spaces induced by nonsingular affine transformations of d-dimensional Euclidean space is investigated. Criteria for their boundedness and estimates for their spectral radii (from above as well as from below) are established. The question of the existence of seminormal composition operators in this class is studied. Cohyponormal composition operators with nontrivial translation part are indicated.

Key words: composition operator, spectral radius, seminormal operator.

Introduction

The foundations of the theory of composition operators in abstract L^2-spaces are well developed. In particular boundedness, subnormality, hyponormality etc. of such operators are completely characterized (cf. [3, 4, 8, 10, 11, 13, 18]). However, if we try to apply directly general theory to concrete classes of composition operators, we get results which are far from being definitive. An attempt to overcome this problem has been done by Mlak in [12] and later by the second-named author in [16].

The present paper, which is an extension and continuation of [16], deals with composition operators on $L^2(\mathbb{R}^d, \mu)$ induced by affine transformations T of $\mathbb{R}^d$, where μ is a positive Borel measure having a radially symmetric density function. Our aim here is to find criteria for their boundedness and to calculate their spectral radii. It turns out that the boundedness of C_T depends only on T and the specific behaviour of μ at infinity (see Theorem 2.2). In general, it is not easy to calculate explicitly the norm of C_T in terms of T and the density function of μ. This is only the case for very particular choices of μ (see Corollary 2.5 and Theorem 5.4). Fortunately, in most cases, we can estimate the norm of C_T and consequently, we can find explicit

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