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Lattice homomorphism — Korovkin type inequalities for vector valued functions

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Abstract. Considered here is the space of continuous functions from a compact and convex subset of a normed vector space into an abstract Banach lattice. Also considered are lattice homomorphisms from the above space into itself or into the associated space of vector valued bounded functions. The uniform convergence of such operators to the unit operator with rates is mainly studied in this article. The produced quantitative results are inequalities which engage the modulus of continuity of the involved continuous function or of its higher order Fréchet derivative.

Key words: Lattice homomorphism, positive operator, Banach lattice, Banach space, modulus of continuity, Fréchet derivatives, unit operator, rate of convergence, Korovkin type inequalities, uniform convergence, continuous function, bounded function.

1. Introduction

The study of the convergence of positive linear operators became more intense and attracted a lot of attention when P. Korovkin (1953) proved his famous theorem (see [8], p. 14).

Korovkin's First Theorem *Let $[a, b]$ be a compact interval in $\mathbb{R}$ and $(L_n)_{n\in\mathbb{N}}$ be a sequence of positive linear operators L_n mapping $C([a, b])$ into itself. Suppose that $(L_n f)$ converges uniformly to f for the three test functions $f = 1, x, x^2$. Then $(L_n f)$ converges uniformly to f on $[a, b]$ for all functions of $f \in C([a, b])$.*

So a lot of authors since then are working on the theoretical aspects of above convergence. But R.A. Mamedov (1959) (see [9]) was the first to put Korovkin's theorem in a quantitative form.

Mamedov's Theorem *Let $\{L_n\}_{n\in\mathbb{N}}$ be a sequence of positive linear operators in the space $C([a, b])$, for which $L_n 1 = 1$, $L_n(t, x) = x + \alpha_n(x)$,*

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