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Some applications of pseudo-differential operators to elasticity

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Abstract. The paper deals with four basic boundary value problem of static elasticity (BPET). It was calculated the principal symbol of a pseudo-differential operator on the boundary whose eigenvalues are the Cosserat eigenvalues of the original BPET. This principal symbol is presented in terms of the principal curvatures and the coefficients of the first quadratic form of the boundary. It was found the principal term in the asympotics of the Cosserat eigenvalues.

Key words: elasticity, isotropic and homogeneous elastic body, Lamé equation, boundary value problems, Poisson constant, pseudo-differential operators, principal symbol, Cosserat spectrum, asymptotics.

Introduction

This paper deals with four basic boundary value problems of static elasticity theory; from now on, we shall call them BPETs. The main tool in our investigation is the calculus of pseudo-differential operators (ΨDO). These methods have been used over the last few years by various authors for investigation both BPET and Stokes problems [3], [6], [7], [11], [12], [13].

Let $\Omega \subset \mathbb{R}^3$ be a bounded domain with an infinitely smooth boundary Γ and let an isotropic, homogeneous elastic body fill Ω. It is well-known that the vector of displacement $u = u(z) = (u_1, u_2, u_3)^t$ satisfies the following Lamé equation (or the Navier equation according to Gurtin [8, p.90]):

$$L_\omega u := \Delta u + \omega \operatorname{grad}\operatorname{div} u = 0, \qquad z \in \Omega \tag{0.1}$$

where Δ is the Laplace operator in $\mathbb{R}^3$, $\omega = (1-2\sigma)^{-1}$ and σ is the Poisson constant. The upper index t denotes the transposition.

Let $g(z) = (g_1, g_2, g_3)^t$ be a given vector-function on Γ, i.e. at $z \in \Gamma$. Let also $N = (N_1, N_2, N_3)^t$ be the inner unit normal vector, τ_1 and τ_2 be orthogonal tangent vectors at each point of boundary Γ (τ_1, τ_2, N form a basis). We shall consider four BPETs following [8], [16, §40] and [14, Chap.3].

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