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Monotoneity and homogeneity of Picard dimensions for signed radial densities

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Abstract. The Picard dimension $\dim P$ of a locally Hölder continuous function P on the punctured unit ball in the d-dimensional Euclidean space ($d \geq 2$) at the origin is the limit of the cardinal number of the set of extremal rays of the cone of nonnegative solutions of the stationary Schrödinger equation $(-\Delta + P(x))u(x) = 0$ on the punctured ball $0 < |x| < a$ with vanishing boundary values on the sphere $|x| = a$ as $a \downarrow 0$. In this paper the monotoneity of $\dim P$ in radial P in the sense that $\dim P \leq \dim Q$ for radial functions P and Q with $P \leq Q$ and the homogeneity of $\dim P$ for radial functions P in the sense that $\dim(cP) \geq \dim P$ $(0 < c \leq 1)$ or equivalently $\dim(cP) \leq \dim P$ $(c \geq 1)$ for radial P are established.

Key words: Picard dimension, Picard principle, Schrödinger equation.

1. Introduction

The purpose of this paper is to contribute to the study on structures of spaces of positive solutions of time independent Schrödinger equations around isolated singularities of their potentials. By translations we may restrict ourselves to the case where isolated singularities of potentials are situated over the origin 0 of the Euclidean space $\mathbf{R}^d$ of dimension $d \geq 2$. Here we denote by Ω_a the punctured ball $0 < |x| < a$ and Γ_a the sphere $|x| = a$ centered at the origin 0 of radius $a > 0$. A real valued locally Hölder continuous function $P(x) = P(x_1, \cdots, x_d)$ defined on $\Omega_a \cup \Gamma_a$ will be referred to as a *density* on $\Omega_a \cup \Gamma_a$, which is viewed as having an isolated singularity at the origin 0, either removable or essential. We consider a stationary Schrödinger equation whose potential is a density $P(x)$ on $\Omega_a \cup \Gamma_a$:

$$(-\Delta + P(x))u(x) = 0 \quad \left(\Delta = \frac{\partial^2}{\partial x_1^2} + \cdots + \frac{\partial^2}{\partial x_d^2}\right). \tag{1}$$

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