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Currents invariant by a Kleinian group

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Abstract. The goal of this paper is to give, under some hypotheses, a characterization of currents and distributions invariant by a group of diffeomorphisms of a manifold M and especially in the case of a Kleinian group Γ acting on the n-sphere $\mathbf{S}^n$.

Key words: current, distribution, Kleinian group, Poincaré exponent, bigraded cohomology.

0. Introduction

Let $p\in\mathbf{N}$ and $\Omega^p(M)$ be the space of differential forms of degree p with compact support in M equipped with its usual C^∞-topology. An element T of the (topological) dual $\mathcal{C}_p(M)$ of $\Omega^p(M)$ is called a *current of degree* p and a *distribution* when $p=0$. An element $T\in\mathcal{C}_p(M)$ is said to be *invariant* (or γ-*invariant*) under the action of a diffeomorphism $\gamma : M\longrightarrow M$ if it satisfies $\langle T,\gamma^*\varphi\rangle=\langle T,\varphi\rangle$ for every $\varphi\in\Omega^p(M)$ or if it vanishes on the space $K^p=\{\varphi-\gamma^*\varphi\ :\ \varphi\in\Omega^p(M)\}$. So the space $\mathcal{C}_p^\Gamma(M)$ (where Γ is the cyclic group generated by γ) of invariant currents on M is canonically isomorphic to the (topological) dual of the quotient $\Omega^p(M)/K^p$. More generally if Γ is a group of diffeomorphisms of M we say that $T\in\mathcal{C}_p(M)$ is Γ-*invariant* if it is invariant by every element $\gamma\in\Gamma$.

In [Ha], Haefliger characterized foliations with minimal leaves in terms of currents invariant by pseudogroups. Thus if the foliation is a suspension with holonomy group Γ, then the interest is focused upon Γ-invariant currents. The case of a Fuchsian group was studied in [HL]: let Γ be a subgroup of the diffeomorphism group Diff($\mathbf{S}^1$) of the circle $\mathbf{S}^1$ whose elements are restriction of elements of a Fuchsian group G of diffeomorphisms of the unit disc $\mathbf{D}$. Suppose that the quotient Riemannian surface $S=G\setminus\mathbf{D}$ is of finite volume, of genus g and with k punctures. Then it was proved in [HL] that *the space of* Γ*-invariant distributions on the circle* $\mathbf{S}^1$ *which vanish on constant functions is isomorphic to the space of harmonic forms on* S *having at most poles of order one at the punctures* x_i. *Its dimension*

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