Hokkaido Mathematical Journal Vol. 26 (1997) p. 157–161

Some solvability criteria for finite groups

Angel CAROCCA[1] and Helder MATOS

(Received December 4, 1995; Revised March 19, 1996)

Abstract. We prove a variant of Kegel-Wielandt's theorem. We use this to give some solvability criteria for factorizable finite groups.

Key words: permutable subgroups, 2-nilpotent subgroups, simple groups.

Let G be a finite group and $G=HK$, where H and K are subgroups of G. There are a number of results which deduce the solvability of G from suitable conditions on H and K. In particular, two of these results are:

Huppert - Itô [4], *If H is supersolvable and K is cyclic of odd order, then $G=HK$ is solvable.*

Kegel - Wielandt ([5], p. 674, Satz 4.3), *If H and K are nilpotent, then $G=HK$ is solvable.*

In the first part of this communication, using classification theorems of simple groups we prove the following:

Theorem A *Let G be a finite group, $H\leq G$ such that $|G:H|=p^{a}$ with p an odd prime number. If H is 2-nilpotent, then G is solvable.*

In the second part, we consider the following definition: A subgroup H of a group G is said to be *semi-normal* in G if there exists a subgroup K of G such that $G=HK$ and H permutes with every subgroup of K.

The solvability of the normal closure of a solvable semi-normal subgroup cannot be concluded in general: In the alternating group $G=\mathbf{A}_{5}$ the subgroups H of index 5 are solvable and semi-normal in G, but the normal closure $H^{G}=G$ is not solvable.

This makes the following theorem interesting:

Theorem B *Let G be a finite group and $H\leq G$ a semi-normal subgroup.*

1991 Mathematics Subject Classification : 20D40, 20D06.

[1]Parcially supported by DIUC/9506j