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Biharmonic green domains in $\boldsymbol{R}^{n}$

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Abstract. The properties of biharmonic functions with a singularity at a finite or infinite point in $\boldsymbol{R}^{n}$, $n\geq 2$, are investigated, leading to a generalization of the classical Bôcher theorem for harmonic functions with positive singularity, when $2\leq n\leq 4$. This latter result is useful in identifying some biharmonic Green domains in $\boldsymbol{R}^{n}$.

Key words: biharmonic point singularities in $\boldsymbol{R}^{n}$.

1. Introduction

The behaviour of a biharmonic function $u(x)$ in $0<|x|<1$ in $\boldsymbol{R}^{n}$, $n\geq 2$, is considered, leading to a necessary and sufficient condition for u to extend as a distribution in $|x|<1$; a case of particular interest is when u is bounded.

The corresponding results when the biharmonic function is defined outside a compact set K in $\boldsymbol{R}^{n}$ lead to an analogue of Bôcher's theorem (after a Kelvin transformation) for positive harmonic functions in $\boldsymbol{R}^{n}\backslash K$; but this is valid only when $2\leq n\leq 4$. A corollary to this is: let Ω be a domain in $\boldsymbol{R}^{n}$, $2\leq n\leq 4$ such that $\boldsymbol{R}^{n}\backslash\Omega$ is compact. Then Ω is not a biharmonic Green domain; that is, a biharmonic Green function cannot be defined on Ω.

2. Preliminaries

For $n\geq 2$, let E_{n} and S_{n} denote the fundamental solutions of the Laplacian Δ and Δ^{2} in $\boldsymbol{R}^{n}$; that is, $\Delta E_{n}=\delta$ and $\Delta^{2}S_{n}=\delta$ in the sense of distributions.

Given a locally integrable function f on $\boldsymbol{R}^{n}$, let $M(r,f)$ denote the mean value of $f(x)$ on $|x|=r$.

Proposition 2.1 *Let $u(x)$ be a harmonic function in $0<|x|<1$ in $\boldsymbol{R}^{n}$. Then the following are equivalent*:

1) *u extends as a distribution in $|x|<1$ (in which case, it is of order*

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