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Triangular forms of subdiagonal algebras

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Abstract. In this note, we study the triangular forms of subdiagonal algebras and discuss the triangular decomposition of subdiagonal algebras.

Key words: von Neumann algebra; expectation; subdiagonal algebra; nest algebra.

1. Introduction and Preliminaries

In [1], Arveson introduced the notion of subdiagonal algebras to give a unified approach to the theory of non-selfadjoint operator algebras. The algebra is not only a noncommutative analogue of weak $*$-Dirichlet algebras but also a generalization of the work of Helson-Lowdenslager in [4]. Thus, the algebra has many analytic properties as the algebra of generalized analytic functions. Several concrete examples were considered in [1]. Further, Loebl-Muhly [7] and Kawamura-Tomiyama [6] gave systematic examples of subdiagonal algebras from the theory of spectral subspaces determined by flows on a von Neumann algebra. We refer the readers to [1] for the elementary properties of subdiagonal algebras. On the other hand, the notion of nest algebras was introduced by Ringrose [8] to study the triangular forms for operators. The structure of nest algebras was studied by many authors and we refer the readers for the details to Davidson's book [3]. Our aim in this note is to study the triangular forms of subdiagonal algebras related to the theory of nest algebras.

At first, we start by giving the definition of subdiagonal algebras. Let $\mathcal{M}$ be a von Neumann algebra on a complex Hilbert space $\mathcal{H}$ and let Φ be a faithful normal positive linear map of $\mathcal{M}$ onto a von Neumann subalgebra $\mathfrak{D}$ of $\mathcal{M}$ which is idempotent, that is, let Φ be a faithful normal expectation of $\mathcal{M}$ onto $\mathfrak{D}$. A subalgebra $\mathfrak{A}$ of $\mathcal{M}$, containing $\mathfrak{D}$, is called a subdiagonal algebra in $\mathcal{M}$ with respect to Φ if

(i) $\mathfrak{A} \cap \mathfrak{A}^* = \mathfrak{D}$;

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