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On the extension properties of Triebel-Lizorkin spaces

(Dedicated to Professor Kyûya Masuda on the occasion of his sixtieth birthday)

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Abstract. Extension of functions originally defined on a domain of a Euclidean space to the whole Euclidean space is considered. Two results on the extension of functions in A. Seeger's generalized Triebel-Lizorkin spaces are proved.

Key words: extension theorem, Triebel-Lizorkin space, (ε, δ)-domain.

1. Introduction

In [S], Seeger introduced some function spaces on domains of $\mathbb{R}^n$, which can be considered as a natural generalization of the Triebel-Lizorkin space to the case of spaces on domains, and gave an extension theorem for those spaces ([*ibid.*; Theorem 2]). The purpose of the present paper is to give some results concerning the extension properties of Seeger's function spaces. In this section, after fixing several notations, we shall state the main results of this paper.

Throughout this paper we use the letters n, Ω, k, p, q, r, and α in the following fixed meanings: n is a positive integer and denotes the dimension of the Euclidean space $\mathbb{R}^n$; Ω denotes an open subset of $\mathbb{R}^n$; k denotes a nonnegative integer; p, q, and r denote positive real numbers or ∞; α denotes a nonnegative real number.

We also use the following notations. The set

$$Q = Q(x,t) = \{(y_i) \in \mathbb{R}^n \mid \max |y_i - x_i| \leqq t\},$$

where $x = (x_i) \in \mathbb{R}^n$ and $0 < t < \infty$, is called a cube with center x and sidelength $2t$. The center of a cube Q is denoted by x_Q and the sidelength by $\ell(Q)$. If $Q = Q(x,t)$ and $0 < a < \infty$, then the cube $Q(x, at)$ is simply denoted by aQ. The Lebesgue measure of a cube Q is denoted by $|Q|$; thus $|Q| = \ell(Q)^n$. A dyadic cube is a cube of the form $\{(x_i) \in \mathbb{R}^n \mid 2^m k_i \leqq x_i \leqq 2^m(k_i + 1),\ i = 1, \dots, n\}$ with m and k_i integers. The set of all dyadic

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