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Decay of solutions to the Cauchy problem for the Klein-Gordon equation with a localized nonlinear dissipation

(Dedicated to Professor Rentaro Agemi on his Sixtieth birthday)

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Abstract. We derive a precise decay estimate of the solutions to the Cauchy problem for the Klein-Gordon equation with a nonlinear dissipation:

$$u_{tt}-\Delta u+u+\rho(x,t,u_t)=0 \quad \text{in} \quad R^N\times[0,\infty),$$

$$u(x,0)=u_0(x) \quad \text{and} \quad u_t(x,0)=u_1(x),$$

where $\rho(x,t,v)$ is a function like $\rho=a(x)(1+t)^{\theta}|v|^r v$, $-1<r$, with $a(x)\ge 0$ supported on $\Omega_R=\{x\in R^N \mid |x|\ge R\}$ for some $R>0$.

Key words: decay, localized dissipation, wave equation.

1. Introduction

In this paper we are concerned with a decay property of the solutions to the Cauchy problem for the Klein-Gordon equation with a dissipative term:

$$u_{tt}-\Delta u+u+\rho(x,t,u_t)=0 \quad \text{in} \quad R^N\times[0,\infty), \tag{1.1}$$

$$u(x,0)=u_0(x) \quad \text{and} \quad u_t(x,0)=u_1(x), \tag{1.2}$$

where $\rho(x,t,v)$ is a function like $\rho=a(x)(1+t)^{\theta}|v|^r v$, $-1<r$, with $a(x)\ge 0$ supported on $\Omega_R=\{x\in R^N|\ |x|\ge R\}$ for some $R>0$.

To explain our problem, let us consider a typical case $\rho=a(x)|v|^r v$.

When $a(x)\ge\varepsilon_0>0$ on R^N, we have proved in [7] that the solution $u(t)\in C^1([0,\infty);L^2(R^N))\cap C([0,\infty);H_1(R^N))$ with $\operatorname{supp}u_0\cup\operatorname{supp}u_1\subset B_L\equiv\{x\in R^N \mid |x|\le L\}$, $L>0$, satisfies the decay estimate

$$E(t)\equiv\frac{1}{2}\{||u_t(t)||^2+||\nabla u(t)||^2+||u(t)||^2\}$$

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