Hokkaido Mathematical Journal Vol. 28 (1999) p. 613–633

Global hypoellipticity of subelliptic operators on closed manifolds

Hideki OMORI and Takao KOBAYASHI

(Received November 25, 1998)

Abstract. We give a criterion of global hypoellipticity on closed manifolds for certain second order operators. Applying this criterion, global hypoellipticity of horizontal Laplacians and an example which has no infinitesimally transitive points are studied.

Key words: global hypoellipticity, horizontal Laplacian.

1. Introduction

A differential operator L on a C^∞ manifold M is *hypoelliptic*, if any distribution solution u of the equation $Lu = f$ is smooth at the place where f is smooth.

Similarly, L is called to be *globally hypoelliptic*, if $Lu \in C^\infty(M)$ for a distribution u implies $u \in C^\infty(M)$. Here $C^\infty(M)$ is the space of smooth functions on M.

It is obvious that if L is hypoelliptic, then L is globally hypoelliptic. In this paper, we concern with the global hypoellipticity of a differential operator on a closed (compact connected without boundary) manifold M.

Let $X_1, \dots, X_m$ be smooth (real) tangent vector fields on M. The differential operator L which we shall treat in this paper is given in the form

$$L := \sum_{i=1}^{m} X_i{}^* X_i, \tag{1.1}$$

where $X_i{}^*$ is the formal adjoint operator of X_i with respect to a fixed smooth Riemannian metric on M. Let $\mathcal{V}$ be the linear space spanned by X_i's

$$\mathcal{V} := \left\{ \sum_{i=1}^{m} f_i X_i : f_i \in C^\infty(M) \right\}, \tag{1.2}$$

1991 Mathematics Subject Classification : 35H05, 58G03.