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On stability of periodic solutions of the Navier-Stokes equations in unbounded domains

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Abstract. We consider the stability of periodic solutions of the Navier-Stokes equations in unbounded domains $\Omega\subset \boldsymbol{R}^{n}$ $(n\geq 3)$, which belong to $BC(\boldsymbol{R};L^{m_{1}}\cap L^{m_{2}})$ for some $n/2<m_{1}<n<m_{2}$. We show that if the periodic solution w is small in $L^{\infty}(0,\infty;L^{m_{1}}\cap L^{m_{2}})$ for some $m_{1}<n<m_{2}$ and if the initial disturbance a is small in $L^{n}(\Omega)$, then w is stable.

Key words: Navier-Stokes equations, unbounded domains, stability, periodic solutions.

1. Introduction

Let Ω be an *exterior* domain in $\boldsymbol{R}^{n}$ $(n\geq 4)$, i.e., a domain having a compact complement $\boldsymbol{R}^{n}\backslash\Omega$, the half space $\boldsymbol{R}_{+}^{n}$ $(n\geq 3)$, or the whole space $\boldsymbol{R}^{n}$ $(n\geq 3)$ and assume that the boundary $\partial\Omega$ is of class $C^{2+\mu}(0<\mu<1)$. The motion of the incompressible fluid occupying Ω is governed by the Navier-Stokes equations:

$$(N-S)\qquad \begin{cases} \dfrac{\partial w}{\partial t}-\Delta w+w\cdot\nabla w+\nabla\pi=f, & \operatorname{div}w=0 \quad x\in\Omega,\ t\in\boldsymbol{R},\\ w=0 \quad \text{on } \partial\Omega, & w(x,t)\rightarrow 0 \quad \text{as } |x|\rightarrow\infty, \end{cases}$$

where $w=w(x,t)=(w^{1}(x,t),\ldots,w^{n}(x,t))$ and $\pi=\pi(x,t)$ denote the unknown velocity vector and the unknown pressure of the fluid, respectively, while $f=f(x,t)=(f^{1}(x,t),\ldots,f^{n}(x,t))$ is the given external force. In [13], Kozono-Nakao constructed periodic strong solutions in unbounded domains for some periodic external force f. Their solutions belong to $BC(\boldsymbol{R};L^{r}\cap L^{\infty})$ for some $n/2<r<n$.

The purpose of the present paper is to show the *stability* of such solutions. If $w(x,0)$ is initially perturbed by a, then the perturbed flow $v(x,t)$ is governed by the following Navier-Stokes equations:

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