Hokkaido Mathematical Journal Vol. 28 (1999) p. 133–145

A construction of closed helices with self-intersections in a complex projective space by using submanifold theory

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(Received January 29, 1998)

Abstract. In this paper we construct a certain class of closed helices with self-intersections in a complex projective plane. This is a nice class of curves which are not generated by global Killing vector fields.

Key words: helix, self-intersection, complex projective space, isometric imbedding, curvature, complex torsion.

Introduction

The main purpose of this paper is to give a certain class of closed helices with self-intersections in a complex projective space by using a well-known isometric imbedding. In a real space form, which is a Euclidean space $\mathbb{R}^n$, a standard sphere S^n, or a hyperbolic space H^n, it is well-known that a smooth curve is a helix if and only if it is generated by a Killing vector field on this space. In a symmetric space of rank one, every circle, that is a helix of order 2, is generated by a Killing vector field. From this point of view we are interested in the difference between the set of helices of order d (≥ 3) and the set of curves generated by Killing vector fields in a symmetric space of rank one. Needless to say, in any Riemannian manifold M every integral curve of a Killing vector field is a helix. But in general, a helix is not necessarily an integral curve of some Killing vector field in M.

One of the most important properties of integral curves of Killing vector fields is that they do not have any self-intersection points. In this paper we pay attention to self-intersection points of helices in a complex projective space. In section 2 we give a class of helices with self-intersection points by using an isometric imbedding of a flat torus into a complex projective plane defined in [4]. This imbedding maps geodesics in a torus to circles

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1991 Mathematics Subject Classification : 53C22, 53C42.

*This research was partially supported by Grand-in-Aid for Encouragement of Young Scientists (No. 09740055), Ministry of Education, Science and Culture