

On the Schur indices of certain irreducible characters of finite Chevalley groups

Zyozyu OHMORI

(Received September 27, 1996; Revised March 23, 1998)

Abstract. Let G be a finite Chevalley group of split type. We shall give some sufficient conditions subject for that G has irreducible characters of the Schur index equal to 2.

Key words: Chevalley groups, irreducible characters, Schur index.

Introduction

Let F_q be a finite field with q elements of characteristic p . Let \mathbf{G} be a connected, reductive algebraic group defined over F_q , and let $F : \mathbf{G} \rightarrow \mathbf{G}$ be the corresponding Frobenius endomorphism of \mathbf{G} . In the following, if H denotes an F -stable subgroup of \mathbf{G} , then the group of F -fixed points of H will be denoted by H . Let \mathbf{B} be an F -stable Borel subgroup of \mathbf{G} , and let \mathbf{U} be the unipotent radical of \mathbf{B} . Then \mathbf{U} is F -stable and \mathbf{U} is a Sylow p -subgroups of \mathbf{G} . According to a theorem of Gel'fand-Graev-Yokonuma-Steinberg, if λ is a linear character of \mathbf{U} in "general position", then the character $\lambda^{\mathbf{G}}$ of G induced by λ is multiplicity-free (see Steinberg [13, Theorem 49, p. 258] and Carter [2, Theorem 8.1.3]). In [5], R. Gow has initiated to study the rationality-properties of the characters $\lambda^{\mathbf{G}}$ where λ runs over certain linear characters of \mathbf{U} and, using the results obtained there, he obtained some informations about the Schur indices of some irreducible characters of G (also cf. A. Helversen-Pasoto [7]). He has treated the case that $\mathbf{G} = GL_n, SL_n$ and Sp_{2n} . In [10], we have obtained some results about the rationality of the $\lambda^{\mathbf{G}}$ when \mathbf{G} is a general reductive group. Our intension here is to get more precise results when \mathbf{G} is a simple algebraic group. The twisted cases are treated in [12]. So, in this paper, we shall treat the untwisted cases. We shall obtain some sufficient conditions subject for that the Schur index of any irreducible character of G is equal to one and some sufficient conditions subject for that G has irreducible characters of the Schur index equal to 2.