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Boundedness of the multiple singular integral operators on product spaces*

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Abstract. In this paper, we consider the $L^{p}(\mathbb{R}^{m}\times\mathbb{R}^{n})$ boundedness for the multiple singular integral operators of Fefferman type, defined by

$$Tf(x_{1},x_{2})=\mathrm{p.v.}\int_{\mathbb{R}^{m}\times\mathbb{R}^{n}}h(|y_{1}|,|y_{2}|)\frac{\Omega(y_{1}',y_{2}')}{|y_{1}|^{m}|y_{2}|^{n}}f(x_{1}-y_{1},x_{2}-y_{2})dy_{1}dy_{2},$$

where $y_{1}\in\mathbb{R}^{m}$, $y_{2}\in\mathbb{R}^{n}$ and $y_{i}'=y_{i}/|y_{i}|$, $h(r,s)$ is bounded on $\mathbb{R}_{+}\times\mathbb{R}_{+}$, Ω satisfies the cancellation condition

$$\int_{S^{m-1}}\Omega(y_{1}',y_{2}')dy_{1}'=\int_{S^{n-1}}\Omega(y_{1}',y_{2}')dy_{2}'=0.$$

We show that if $\Omega\in L(\log^{+}L)^{2}(S^{m-1}\times S^{n-1})$, then T is bounded on $L^{p}(\mathbb{R}^{m}\times\mathbb{R}^{n})$ for all $1<p<\infty$.

Key words: multiple singular integral operator, Fourier transform estimate, Littlewood-Paley theory.

1. Introduction and Statement of the Result

Let $h(r,s)$ be a bounded function on $\mathbb{R}_{+}\times\mathbb{R}_{+}$, and $\Omega(y_{1},y_{2})$ a function defined on $S^{m-1}\times S^{n-1}$ $(m,n\geq 2)$ satisfying

$$\int_{S^{m-1}}\Omega(y_{1}',y_{2}')dy_{1}'=\int_{S^{n-1}}\Omega(y_{1}',y_{2}')dy_{2}'=0,\tag{1}$$

where S^{m-1} (resp. S^{n-1}) is the unit sphere of $\mathbb{R}^{n}$ (resp. $\mathbb{R}^{m}$). For $y\in\mathbb{R}^{m}$, let $y'=y/|y|$. Define the multiple singular integral operator

$$Tf(x_{1},x_{2})=\mathrm{p.v.}\int_{\mathbb{R}^{m}\times\mathbb{R}^{n}}h(|y_{1}|,|y_{2}|)\frac{\Omega(y_{1}',y_{2}')}{|y_{1}|^{m}|y_{2}|^{n}}f(x_{1}-y_{1},x_{2}-y_{2})dy_{1}dy_{2}.\tag{2}$$

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