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Moser type theorem for toric hyperKähler quotients

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Abstract. We consider the symplectic geometry of toric hyperKähler quotients. Under a mild condition, we obtain that toric hyperKähler quotients have stability about its underlying symplectic structures.

Key words: hyperKähler quotients, symplectic manifolds, Hamiltonian torus actions, Moser theorem.

1. Introduction

Symplectic manifolds have the properties of both softness and hardness. For softness, there is a classical theorem due to Moser [6].

Theorem 1.1 (Moser) *Let M be a closed manifold and $\{\omega_t\}_{0\leq t\leq 1}$ a smooth family of cohomologous symplectic forms on M. Then there exists $\{\phi_t\}_{0\leq t\leq 1}$ a smooth family of diffeomorphisms of M such that $\phi_t^*\omega_t=\omega_0$ for all $t\in[0,1]$.*

This theorem is proved by constructing a family of vector fields $\{Z_t\}_{0\leq t\leq 1}$ whose integral flows induce $\{\phi_t\}_{0\leq t\leq 1}$. Therefore the completeness of these vector fields is necessary. But this is automatically satisfied since M is compact. In this paper, we prove that an analog of this theorem holds in the case of not necessarily compact but complete hyperKähler quotients under a mild condition.

Let (M,g,I,J,K) be a complete hyperKähler manifold, i.e. g is a complete Riemannian metric and I, J, K are almost complex structures of M satisfying

(i) g is Hermitian with respect to I, J, K,
(ii) $I^2=-1$, $J^2=-1$, $K^2=-1$, $IJ=K$, $JK=I$, $KI=J$,
(iii) $\nabla I=0$, $\nabla J=0$, $\nabla K=0$,

where ∇ is the Levi-Civita connection of g.

We define the 2-forms ω_I, ω_J, ω_K by $\omega_I(X,Y)=g(IX,Y)$, etc. From the condition above, it follows that I, J, K are integrable and ω_I, ω_J, ω_K

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