

## Sheaves on the category of periodic observation

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**Abstract.** A Grothendieck topology on the subgroup category of the additive group of integers is defined and the sheafification of the presheaves induced from discrete dynamical systems are determined.

*Key words:* category, dynamical systems, Grothendieck topology, sheafification.

### Introduction

Suppose various observers record the activity of one object periodically with their own time units and each obtains his own dynamical model of the object. How should we obtain a comprehensive model of the object starting from these personal models?

This question may be regarded as a special case of the universal problem of recovering the global information from coherent pieces of local information, which is often analyzed succinctly by the sheaf theory.

In this paper, we introduce a Grothendieck topology on the category of observers with different time units and show that the sheafification procedure gives us an effective method of synthesizing the personal dynamical models of observers whose time units generates the unit ideal of the integer.

### 1. The category of observers with different time units

#### 1.1. The category $\mathbf{N}^\circ$

Let  $\mathbf{N}^\circ$  be the category whose objects are natural integers and whose arrows are generated by  $\{ \beta_{n,m} : m \rightarrow n \mid n|m \}$  and  $\{ \alpha_n : n \rightarrow n \mid n \in \mathbf{N} \}$  with the following relations:

$$\begin{aligned}\beta_{nn} &= 1_n \\ \beta_{\ell,m}\beta_{m,n} &= \beta_{\ell,n}\end{aligned}$$