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On Littlewood-Paley functions and singular integrals

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Abstract. In this paper, we obtain certain sufficient conditions for the L^{p} boundedness on Littlewood-Paley functions and on some singular integrals. As applications, we study Marcinkiewicz integrals and singular integrals whose kernels are rough not only on the sphere, but also in the radial direction.

Key words: Littlewood-Paley theory, g-function, singular integral.

1. Introduction

Let $\mathbb{R}^{n}$ be n-dimensional Euclidean space and $\mathbb{T}^{n}$ be the n-dimensional torus. $\mathbb{T}^{n}$ can be identified with $\mathbb{R}^{n}/\Lambda$, where Λ is the unit lattice which is the additive group of points in $\mathbb{R}^{n}$ having integral coordinates. For an $L^{1}(\mathbb{R}^{n})$ function Φ we define $\Phi_{t}(x)=2^{-tn}\Phi(x/2^{t})$, $t\in \mathbb{R}$. Then the Fourier transform of Φ_{t} is $\widehat{\Phi}_{t}(\xi)=\widehat{\Phi}(2^{t}\xi)$. The Littlewood-Paley g-function $g_{\Phi}(f)$ on $\mathbb{R}^{n}$ is defined by

$$g_{\Phi}f(x)=\left(\int_{\mathbb{R}}|\Phi_{t}*f(x)|^{2}dt\right)^{1/2}, \tag{1.1}$$

initially, for f in the Schwartz space $\mathcal{S}(\mathbb{R}^{n})$.

The Littlewood-Paley g-function on $\mathbb{T}^{n}$ can be defined similarly. For $\widetilde{f}\in C^{\infty}(\mathbb{T}^{n})$, $\widetilde{f}$ has the Fourier series

$$\widetilde{f}(x)=\sum_{k\in\Lambda}a_{k}e^{2\pi i\langle k,x\rangle}.$$

We let

$$G_{\Phi}\widetilde{f}(x)=\left(\int_{\mathbb{R}}|\widetilde{\Phi}_{t}*\widetilde{f}(x)|^{2}dt\right)^{1/2} \tag{1.1'}$$

where

$$\widetilde{\Phi}_{t}*\widetilde{f}(x)=\sum_{k\in\Lambda}\widehat{\Phi}(2^{t}k)a_{k}e^{2\pi i\langle k,x\rangle}.$$

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