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On the Schur indices of the irreducible characters of $SL(n, q)$

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Abstract. We shall give some sufficient conditions subject for that the Schur indices of irreducible characters of the special linear groups over finite fields are equal to one.

Key words: special linear groups, irreducible characters, Schur index.

Introduction

Let S denote the special linear group $SL(n, q)$ of degree $n \geqq 2$ over a finite field $\mathbb{F}_q$ with q elements of characteristic p. If χ is a complex irreducible character of a finite group and K is a field of characteristic 0, then $m_K(\chi)$ denotes the Schur index of χ with respect to K, where we consider χ as a character over some algebraically closed extension of K. Then the following results are known:

Theorem A (R. Gow [3]) *For any (complex) irreducible character* χ *of* S, *we have* $m_{\mathbb{Q}}(\chi) \leqq 2$.

Theorem B (A.V. Zelevinsky [15]) *Assume that* $p = 2$. *Then, for any irreducible character* χ *of* S, $m_{\mathbb{Q}}(\chi) = 1$.

Theorem C (Z. Ohmori [9]) *Assume that* $p \neq 2$ *and* n *is odd. Then, for any irreducible character* χ *of* S, $m_{\mathbb{Q}}(\chi) = 1$.

Theorem D (Gow [3]) *Assume that* $p \neq 2$, *n is even, and* $\mathrm{ord}_2 n > \mathrm{ord}_2(p-1)$. *Then, for any irreducible character* χ *of* S, $m_{\mathbb{Q}}(\chi) = 1$.

Theorem E (Gow [3]) *Assume that* $p \neq 2$, *n is even,* $\mathrm{ord}_2 n \leqq \mathrm{ord}_2(p-1)$, *and q is an even power of p. Let* χ *be any irreducible character of S. Then, if* $\chi(-1_n) = \chi(1_n)$, *we have* $m_{\mathbb{Q}}(\chi) = 1$. *If* $\chi(-1_n) = -\chi(1_n)$, *then, for any prime number* $r \neq p$, *we have* $m_{\mathbb{Q}_r}(\chi) = 1$.

Theorem F (Gow [3]) *Assume that* $p \neq 2$ *and* $n = 4m$ *for some positive*

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