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A generalization of prime graphs of finite groups

Seiichi ABE and Nobuo IIYORI

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Abstract. We investigate some properties of generalized prime graphs of finite groups, especially solvable graphs which is newly defined in this paper. A solvable graph of a finite simple group has a striking feature, that is, connected and incomplete. In the last section we give some applications of solvable graphs. We use the classification of finite simple groups in order to prove main theorems.

Key words: Sylow subgroup, prime graphs, simple groups.

1. Introduction

There are a lot of ways to characterize a finite group by orders of its elements. Considering a prime graph is one of such ways. In a prime graph $\Gamma(G)$ of a finite group G, edges p and q are defined to be joined when there exists an element x of G whose order is pq. This condition can be interpreted that G includes a cyclic subgroup of order pq. So it seems natural to consider some other graphs in which the condition "being cyclic" is replaced to other ones. We will discuss solvable graphs which will be defined afterward of this paper and will show some applications of the graphs. Every group appearing in this paper is a finite group. Following the notation in Iiyori-Yamaki [4] and Williams [9], π_i stands for the ith connected components of prime graphs in tables of [4, 9] and we let $\mathrm{com}(G)$ stand for the number of connected components of prime graph of G.

2. Definitions and Remarks

Definition 1 Let Λ be a set of positive rational integers. We denote Λ-graph by Γ_Λ and the set of vertices of Γ_Λ by V_Λ which is the set of primes which divide an element of Λ. For vertices p and q of Γ_Λ, p is joined to q if and only if there exists an element a in Λ such that $pq|a$.

For example, let $\Lambda = \{6, 7, 30, 33\}$. Then Γ_Λ is the following;

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