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Putnam's theorems for w-hyponormal operators

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Abstract. Three theorems on hyponormal operators due to Putnam are generalized to apply to the broader class of w-hyponormal operators. In particular, it is shown that if an operator T is w-hyponormal and the spectrum of $|T^*|$ is not an interval, then T has a nontrivial invariant subspace.

Key words: p-, log- and w-hyponormal operators, approximate point spectrum, invariant subspace.

1. Introduction

Let T be a bounded linear operator on a Hilbert space H with inner product $(\cdot,\cdot)$ and $p>0$. The operator T is said to be p-hyponormal if $(T^*T)^p \geq (TT^*)^p$. A p-hyponormal operator is said to be hyponormal if $p=1$, semi-hyponormal if $p=1/2$. It is a consequence of the well-known Löwner-Heinz inequality that if T is p-hyponormal, then it is q-hyponormal for any $0<q\leq p$. An invertible operator T is said to be log-hyponormal if $\log|T| \geq \log|T^*|$. Clearly, every invertible p-hyponormal operator is log-hyponormal. Let $T=U|T|$ be the polar decomposition of the operator T. Following [1], we define $\widetilde{T}=|T|^{1/2}U|T|^{1/2}$. An operator T is said to be w-hyponormal if

$$|\widetilde{T}| \geq |T| \geq |\widetilde{T}^*|. \tag{1.1}$$

Inequalities (1.1) show that if T is w-hyponormal, then $\widetilde{T}$ is semi-hyponormal. The classes of log- and w-hyponormal operators were introduced and their spectral properties studied in [2]. It was shown in [2] and [3] that the class of w-hyponormal operators contains both the p- and log-hyponormal operators. Log-hyponormal operators were independently introduced by Tanahashi in the paper [8]. There he gave an example of a log-hyponormal operator which is not p-hyponormal for any $p>0$. Thus, neither the class of p-hyponormal operators nor the class of log-hyponormal operators contains the other. In [4], we pointed out that if T is the

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