

An abstract degenerate hyperbolic equation with application to mixed problems

Piero D'ANCONA and Mariagrazia Di FLAVIANO

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Abstract. We prove an existence result for the Cauchy problem associated to an abstract degenerate hyperbolic equation. Moreover we show several applications to mixed initial boundary value problems for weakly hyperbolic equations.

Key words: nonlinear weakly hyperbolic equations, abstract equations, degenerate equations, mixed initial boundary value problem.

1. Introduction

Let H be a Hilbert space with norm $|\cdot|$, and $\mathbf{B} = (B_1, \dots, B_n)$ an n -tuple of *selfadjoint* operators on H , with (dense) domains $D(B_j)$. For any multiindex $\alpha = (\alpha_1, \dots, \alpha_n)$ and integer s we shall use the notation

$$\mathbf{B}^\alpha = B_1^{\alpha_1} \circ \dots \circ B_n^{\alpha_n}.$$

The subspaces H^s are defined as follows:

$$H^s = \bigcap_{1 \leq j_i \leq n} D(B_{j_1} \circ \dots \circ B_{j_s}).$$

We can obviously endow H^s with a Hilbert space structure with norm

$$|u|_s^2 = \sum_{\substack{0 \leq k \leq s \\ 1 \leq j_i \leq n}} |B_{j_1} \circ \dots \circ B_{j_k} u|^2.$$

We shall solve the following Cauchy problem on H :

$$u'' + \sum_{|\alpha|=2m} a_\alpha(t) \cdot \mathbf{B}^\alpha u = f(t) \tag{1.1}$$

$$u(0) = u_0, \quad u'(0) = u_1. \tag{1.2}$$

We shall assume that

$$\text{the functions } a_\alpha(t) \text{ are real analytic on } [0, T] \tag{1.3}$$