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Solvability of convolution equations in $\mathcal{D}'_{L^p}$

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Abstract. In this paper we give a necessary condition on the Fourier transform of a convolution operator S of the space $\mathcal{D}'_{L^p}; 2 \leq p < \infty$, for the equation $S * u = v$ to have a solution u in $\mathcal{D}'_{L^p}$ for every v in $\mathcal{D}'_{L^p}$. In the case $p = 2$, this condition with the additional assumption $\widehat{S}(\xi) \neq 0$ for all $\xi \in \Re^n$, are sufficient for solvability of the convolution equation.

Key words: distributions of L^p-growth, convolution equations.

1. Introduction

Convolution equations in spaces of distributions and ultradistributions of L^p-growth were studied by several authors. In this work we study the problem of characterizing the convolution operators S for which the convolution equation $S * u = v$ have a solution u in $\mathcal{D}'_{L^p}$ for every v in $\mathcal{D}'_{L^p}$. Pahk [3] characterized hypoelliptic convolution operators in the space $\mathcal{D}'_{L^\infty}$, and left the problem of solvability of convolution equations in $\mathcal{D}'_{L^p}$, $1 \leq p \leq \infty$ open. Pilipovič [4] has established necessary condition and sufficient condition on the convolution operator S to be invertible in $\mathcal{D}'^{(M_p)}_{L^2}$. Moreover, Pilipovič characterized hypoelliptic convolution operators in $\mathcal{D}'^{(M_p)}_{L^2}$. Here we give a necessary condition on $\widehat{S}$, the Fourier transform of the convolution operator S, for the convolution equation $S * u = v$ to have a solution u in $\mathcal{D}'_{L^p}$ for a given v in $\mathcal{D}'_{L^p}$. Moreover, in the case $p = 2$ we give sufficient conditions for solvability of the equation $S * u = v$. Characterizing invertible and hypoelliptic convolution operators in $\mathcal{D}'_{L^p}$ is difficult in general. This is due to lack of differentiability of $\widehat{S}$. It is known (see [1] part (c) of Theorem 2 and the remark which follows it on page 202) that the Fourier transform of any convolution operator in $\mathcal{D}'_{L^p}, 1 \leq p \leq \infty$, is a continuous function which is slowly increasing at infinity. We remark that in this work

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