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On semisimple extensions of serial rings

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Abstract. We prove that if B is a commutative local serial ring and A is a B-algebra which is a left semisimple extension of B, A is a uniserial ring. If in addition A is indecomposable as ring, the lengths of the composition serieses of Ae and B are same for each primitive idempotent e of A. We also give some necessary and sufficient conditions for A to be a left semisimple extension of a subring B of it, in the case where A and B are local serial rings or the case where B is a commutative local serial ring and A is a B-algebra which is serial.

Key words: serial ring, uniserial ring, composition series, semisimple extension.

Throughout this paper A will always be a ring with identity 1, and B a subring of A containing 1. In their previous paper [4] the authors introduced the notion of semisimple extensions of a ring. A ring A is said to be a left semisimple extension of B in the case where every left A-module M is (A, B)-projective, that is, the map π of $A \otimes_B M$ to M, defined by $\pi(a \otimes m) = am$ for any $a \in A$ and $m \in M$, splits as left A-homomorphism, or equivalently, for every left A-module M, every A-submodule which is a B-direct summand of M is always an A-direct summand. (See Theorem 1.1 [4]). The right semisimple extension is similarly defined, and the both left and right semisimple extension is called semisimple extension. Till now some typical examples of the semisimple extension are known, for example, each semisimple ring is a semisimple extension of each subring of it, and each separable extension is a semisimple extension. However, since the semisimplicity is a quite abstract condition, it is very difficult to research the structure of the semisimple extension or find proper examples of it.

In this paper we will give some structure theorem of semisimple extensions of (two-sided) uniserial local rings. A ring R is said to be left serial in the case where R is left artinian and Re has the unique composition series for each primitive idempotent e of R. In the case where R is a direct sum of finite primary left serial rings, R is said to be a left uniserial ring. It is

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