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Root separation on generalized lemniscates[1]

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Abstract. We discuss several positivity type criteria for a polynomial to have all the roots inside, or outside certain planar semi-algebraic domains. The main examples of such domains are the quadrature domains for analytic functions. Compared to the classical separation results for the disk or half-plane, in this more general setting a number of free parameters enter into the positivity criteria. We also remark that the complements on the Riemann sphere of these semi-algebraic domains are appropriate for solving bounded analytic interpolation problems.

Key words: root separation, quadrature domain, positive definite matrix, bounded analytic interpolation.

1. Introduction

The prototype for the class of planar domains which make the object of this note is a quadrature domain for complex analytic functions. We first define the latter, more restrictive family of domains.

Let dA denote the planar Lebesgue measure. Following Aharonov and Shapiro [2] a bounded domain Ω of the complex plane is called a *quadrature domain* if there exists a finitely supported distribution u on the complex plane such that $\mathrm{supp}(u) \subset \Omega$ and the following Gaussian type quadrature formula:

$$\int_{\Omega} f dA = u(f), \qquad f \in L_a^1(\Omega),$$

holds for all integrable, complex analytic functions f in Ω. For instance a disk $D(a, r)$ satisfies such a formula with $u(f) = \pi r^2 f(a)$. This class of domains turns out to have many remarkable properties and connections to several areas of mathematics. For an account of their theory we refer to the monograph [16], the papers [2], [6], [7], [8] and the references cited there.

For the purposes of this note we need only know that the boundary of a quadrature domain Ω is real algebraic (and irreducible) given by an

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