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Examples of compact Toeplitz operators on the Bergman space

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Abstract. R. Yoneda studied compact Toeplitz operators on the Bergman space for special symbols and he posed several problems. In this paper, we give counterexamples for some of these problems.

Key words: Bergman space, Toeplitz operator, compact operator.

1. Introduction

Let D be the open unit disc in the complex plane $\mathbb{C}$. Let dA be the normalized area measure on D. The Bergman space on D, denoted by $L_a^2(D)$, is the space of analytic functions f on D such that

$$\|f\|^2 = \int_D |f(z)|^2 dA(z) < \infty.$$

Let P be the orthogonal projection from $L^2(D, dA)$ onto $L_a^2(D)$. For ϕ in $L^\infty(D)$ the Toeplitz operator $T_\phi : L_a^2(D) \to L_a^2(D)$ is defined by $T_\phi f = P(\phi f)$, $f \in L_a^2(D)$. Put

$$k_z(w) = \frac{1-|z|^2}{(1-\bar{z}w)^2} \quad \text{for} \quad z, w \in D,$$

and k_z is called the normalized reproducing kernel for z. For $z \in D$, define

$$\varphi_z(w) = \frac{z-w}{1-\bar{z}w}, \quad w \in D.$$

It is known several characterization for the compactness of T_ϕ. In [5, Theorem 4], Zheng proved the next theorem.

Theorem A *Let ϕ be in $L^\infty(D)$. Then the following are equivalent.*
(i) *T_ϕ is a compact operator on $L_a^2(D)$.*
(ii) *$\|T_\phi k_z\| \to 0$ as $|z| \to 1-$.*

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