Hokkaido Mathematical Journal Vol. 30 (2001) p. 689–695

Note on Miyashita-Ulbrich action and H-separable extension

Lars KADISON

(Received July 24, 2000)

Abstract. The Miyashita-Ulbrich action is an action of a Hopf algebra A on the centralizer E^C associated to an A-Galois extension B/C with algebra homomorphism α : $B \to E$. Doi and Takeuchi [DT] ask when the action of a Hopf algebra A on the centralizer E^C of a ring extension E/C comes from such an A-Galois extension B/C. They provide an affirmative answer for Azumaya algebra E with subalgebra C such that E_C is a progenerator. In this note we observe how their proof extends to an H-separable extension E/C with the same condition on E_C. Similarly, we establish the converse: if E/C is an H-separable, right A-Galois extension, then E^C is a left A^*-Galois extension over the center $Z(E)$.

Key words: Hopf-Galois extension, centralizer, Miyashita-Ulbrich action, H-separable extension, Azumaya algebra.

1. Introduction

We let R be a commutative ground ring, A be a Hopf algebra over R which is finite projective as an R-module, and A^* its dual Hopf algebra. If B is an associative unital algebra and M is a unitary B-bimodule, we let M^B denote the central elements of M : $M^B = \{m \in M \mid mb = bm, \ \forall b \in B\}$. Ulbrich [U] defines an action of A on the centralizer $V := V_B(C) = B^C$ of an A-Galois extension B/C. If $\beta : B \otimes_C B \to B \otimes A$ denotes the Galois isomorphism given by $\beta(x \otimes y) = xy_{(0)} \otimes y_{(1)}$, the action of an $a \in A$ on x in the centralizer V is given by

$$x \triangleleft a = \sum_i b_i x b'_i \tag{1}$$

where $\sum_i b_i \otimes b'_i = \beta^{-1}(1 \otimes a)$. It is easy to compute that this is a module action with invariant subalgebra $Z(B)$, the center of B. Indeed, it is a measuring action with V becoming a right A-module algebra [U, II].

Doi and Takeuchi [DT] extend this action to the centralizer E^C of an algebra extension E/C with algebra homomorphism $\alpha : B \to E$ by means

2000 Mathematics Subject Classification : 12F10, 16W30.