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On isomorphy of pure hulls of purifiable torsion-free subgroups

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Abstract. A subgroup A of an arbitrary abelian group G is said to be *purifiable* in G if there exists a pure subgroup H of G containing A which is minimal among the pure subgroups of G that contain A. Such a subgroup H is said to be a *pure hull* of A in G. In general, not all pure hulls of purifiable subgroups of arbitrary abelian groups are isomorphic. We show that if A is a purifiable torsion-free subgroup of an arbitrary abelian group, then all pure hulls of A are isomorphic and for all pure hulls H of A, the quotients H/A are isomorphic.

Key words: purifiable subgroup, pure hull, almost-dense subgroup.

All groups considered are arbitrary abelian groups. The terminologies and notations not expressly introduced here follow the usage of [6]. Throughout this note, p denotes a prime integer, G_p the p-primary subgroup and T the maximal torsion subgroup of the abelian group G.

Definition 1 A subgroup A of an abelian group G is said to be *purifiable* in G if, among the pure subgroups of G containing A, there exists a minimal one. Such a minimal pure subgroup is called a *pure hull* of A.

Hill and Megibben [8] established properties of pure hulls of p-groups and characterized the p-groups for which all subgroups are purifiable.

Later, Benabdallah and Irwin [2] introduced the concept of almost-dense subgroups of p-groups and used it to characterize pure hulls of purifiable subgroups of p-groups.

Furthermore, Benabdallah and Okuyama [3] introduced new invariants, the so-called *n-th overhangs* of a subgroup of a p-group, which are related to the n-th relative Ulm-Kaplansky invariants. Using them, they obtained a necessary condition for subgroups of p-groups to be purifiable.

Benabdallah, Charles, and Mader [1] introduced the concept of maximal vertical subgroups supported by a given subsocle of a p-group and

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