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Local equivalence of Sacksteder and Bourgain hypersurfaces

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Abstract. Finding examples of tangentially degenerate submanifolds (submanifolds with degenerate Gauss mappings) in an Euclidean space R^4 that are noncylindrical and without singularities is an important problem of differential geometry. The first example of such a hypersurface was constructed by Sacksteder in 1960. In 1995 Wu published an example of a noncylindrical tangentially degenerate algebraic hypersurface in R^4 whose Gauss mapping is of rank 2 and which is also without singularities. This example was constructed (but not published) by Bourgain.

In this paper, the authors analyze Bourgain's example, prove that, as was the case for the Sacksteder hypersurface, singular points of the Bourgain hypersurface are located in the hyperplane at infinity of the space R^4, and these two hypersurfaces are locally equivalent.

Key words: Gauss mapping, varieties with degenerate Gauss mappings, hypercubic, Sacksteder, Bourgain.

1. It is important to find examples of tangentially degenerate submanifolds in order to understand the theory of such manifolds. These examples prove the existence of tangentially degenerate submanifolds and help to illustrate the theory. The first known example of a tangentially degenerate hypersurface of rank 2 without singularities in R^4 was constructed by Sacksteder [S60]. This example was examined from the differential geometry point of view by Akivis in [A87]. In particular, Akivis proved that the Sacksteder hypersurface has no singularities since they "went to infinity". In the same paper, Akivis presented a series of examples generalizing Sacksteder's example in R^4, constructed a new series of examples of three-dimensional submanifolds $V^3 \subset P^n(\mathbb{R})$, $n \geq 4$, of rank 2, whose focal surfaces are imaginary, and proved existence of submanifolds of this kind. Note that more examples of tangentially degenerate submanifolds without singularities can be found in [I98, I99a, I99b]. The examples are essentially based on classical Cartan's hypersurfaces (see [C39]).

Mori [M94] claims that he constructed "a one-parameter family of com-

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