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# The discrete and nondiscrete subgroups of $SL(2, R)$ and $SL(2, C)$*

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**Abstract.** It is shown in this paper that some special subsets of $\mathrm{tr}(G)=\{\mathrm{tr}(f): f\in G\}$ are sufficient to determine whether $G$ is discrete or not when $G\subset SL(2, R)$ or $SL(2, C)$ is nonelementary. One of Beardon's open problems is affirmatively answered.

*Key words*: discreteness, nondiscreteness, trace.

## 1. Introduction

In [1], Beardon tried to determine whether a group $G$ of $SL(2, R)$ is discrete or not by using some subset of $\mathrm{tr}(G)=\{\mathrm{tr}(f): f\in G\}$ when $G$ is finitely generated and contains parabolic elements.

The main aim of this paper is twofold. The first is to generalize Beardon's discussion as mentioned above. Our main results are Theorems 3.1 and 3.2. The second is to give an affirmative answer to one of the open problems raised by Beardon in [1].

## 2. Some concepts and notations

Let $G$ be a subgroup of $SL(2, C)$. $G$ is called elemetary if $G$ has a finite orbit in $\bar{H}^3$, i.e., there is $z\in\bar{H}^3$ such that $G_z=\{\widetilde{f}(z): f\in G\}$ is finite, where $\widetilde{f}$ denotes the Poincaré extension of $f$ (cf. [2]). Otherwise $G$ is called nonelementary.

From [7], the following is obvious.

**Lemma 2.1** *Two elliptic elements $f, g\in SL(2, R)$, whose orders are not both equal to 2, generate a nonelementary group if and only if $f$ and $g$ have no common fixed points in $\bar{C}$.*

For any nonelementary group $G\in SL(2, R)$, let

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