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A modified Newton method for asymmetric variational inequality problems

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Abstract. Based on the regularized gap function introduced by Fukushima [5], we present a modification of Newton's method for solving the variational inequality problem $VI(X, F)$ by combining the trust region technique. Without the assumption that the mapping F is strongly monotone on the set X, we prove that the proposed algorithm converges globally to a solution of $VI(X, F)$ if the Jacobian matrix ∇F is positive definite on X. Under some additional assumptions, we deduce that the rate of convergence is quadratic.

Key words: variational inequality problem, Newton's method, trust region technique, global convergence, quadratic convergence.

1. Introduction

We consider the variational inequality problem (denoted by $VI(X, F)$): find an $x^* \in X$ such that

$$\langle F(x^*), x - x^*\rangle \geq 0, \quad \text{for all} \quad x \in X,$$

where $X \subseteq R^n$ is a nonempty closed convex set, $F : R^n \to R^n$ is a continuous mapping and $\langle\cdot,\cdot\rangle$ denotes the inner product in R^n. In the special case where $X = R_+^n$, the variational inequality problem reduces to the nonlinear complementarity problem.

The variational inequality problem has many applications in economics, engineering and various equilibrium models. In the last several years, there have been developed many numerical methods for solving the variational inequality problem. We refer to [7] for a comprehensive review about the early developments of such approaches. They are mainly divided into two classes: one is to reformulate the variational inequality problem as a system of equations and then to use methods and theory from the classical system of equations, such as projection methods, the nonlinear Jacobi method, SOR method, generalized gradient method, Newton's method and quasi-

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