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# On the scaling exponents of Takagi, Lévy and Weierstrass functions

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**Abstract.** We study the scaling exponents of the Takagi, Lévy and Weierstrass functions. We show that their pointwise Hölder exponents coincide with their weak scaling exponents at each point of the real line. A partial result about the scaling exponent of the Lévy function is also given.

*Key words*: wavelets, scaling exponents, Takagi function, Lévy function, Weierstrass functions.

## 1. Introduction

Let $s$ be a positive number, which is not an integer and let $x_0$ be a point in $\mathbf{R}^n$. Then a function $f$ on $\mathbf{R}^n$ belongs to the pointwise Hölder space $C^s(x_0)$, if there exists a polynomial $P$ of degree less than $s$ such that

$$|f(x)-P(x-x_0)|\leq C|x-x_0|^s$$

in a neighborhood of $x_0$. $\mathcal{S}_0(\mathbf{R}^n)$ denotes the closed subspace of the Schwartz class $\mathcal{S}(\mathbf{R}^n)$ such that

$$\int_{\mathbf{R}^n} x^{\alpha}\psi(x)\,dx=0$$

for any multi-index $\alpha$ in $\mathbf{Z}_+^n$. Then a tempered distribution $f$ belongs to $\Gamma^s(x_0)$, if for each $\psi$ in $\mathcal{S}_0(\mathbf{R}^n)$, there exists a constant $C(\psi)$ such that

$$\left|\int_{\mathbf{R}^n} f(x)\frac{1}{a^n}\psi\left(\frac{x-x_0}{a}\right)dx\right|\leq C(\psi)a^s,\quad 0<a\leq 1.$$

Let $\varphi$ be a function in the Schwartz class $\mathcal{S}(\mathbf{R}^n)$ such that

$$\hat{\varphi}(\xi)=\begin{cases}1 & \text{on } |\xi|\leq\dfrac{1}{2}\\ 0 & \text{on } |\xi|\geq 1\end{cases},$$

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