

Singularities for projections of contour lines of surfaces onto planes

Yasuhiro KUROKAWA

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Abstract. We study semi-local patterns of the visions for contour lines of a surface when one looks at it from a distant view in any direction. The study of such a landscape (i.e. so-called “topography”) is reduced to the study of a certain divergent diagram of smooth mappings $\mathbb{R} \leftarrow M \rightarrow \mathbb{R}^2$, where M is a smooth surface. We give a generic semi-local classification of such divergent diagrams.

Key words: singularity, vision, divergent diagram, semi-local, web structure.

1. Introduction

A contour line of a surface S in $\mathbb{R}^3 = \{(x, y, z)\}$ is a set $S \cap \{z = \text{constant}\}$. In this paper we study semi-local patterns of the viewing image (i.e. topography) when one looks at a surface with its contour lines from a distant point in \mathbb{R}^3 . We give a generic semi-local classification for the singularities of orthogonal projections of contour lines of surfaces onto the plane.

There are many studies of certain visual images from the viewpoint of singularity theory started by Koenderink and Doorn [KD] (see also [K], [W], [B], [BG], [P], [A], [DT]). In particular Dufour and Tueno [DT] have investigated local and semi-local generic types of photographs (i.e. equal illumination curves) of lighted surfaces. In [DT] the generic classification of divergent diagrams $(\mathbb{R}, 0) \leftarrow (\mathbb{R}^2, 0) \rightarrow (\mathbb{R}^2, 0)$ due to Arnol’d [A] and Dufour [D4] was applied. However normal forms of the semi-local case were not given.

In this paper, we give more detailed classification of topographies in the semi-local case. In particular, we give the normal forms of generic types, which contain “functional moduli”. These results will be used to show the so-called “topological rigidity theorem” in the semi-local case of

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