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Semilinear heat equations with distributions in Morrey spaces as initial data

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Abstract. This paper is a continuous study to the paper [27]. Here we consider in Morrey spaces the Cauchy problem of the general semilinear heat equation with an external force. Both the external force and initial data belong to suitable Morrey spaces. When the norm of the external force is small, we proved the unique existence of small solution to the corresponding stationary problem. Moreover, if the initial data is close enough to the stationary solution, we verified the time-global solvability of the Cauchy problem, which leads to the stability of the small stationary solution.

Key words: semilinear heat equations, Morrey spaces, semigroup.

1. Introduction

Let us consider the Cauchy problem of the following semilinear heat equation with an external force $f(x)$ in $\mathbf{R}^n$ for $n \geq 3$:

$$\frac{\partial v}{\partial t}(t,x) = \Delta v(t,x) + v(t,x)|v(t,x)|^{\nu-1} + f(x) \quad \text{in} \quad (0,\infty)\times\mathbf{R}^n, \tag{1.1}$$

$$v(0,x) = a(x) \quad \text{on} \quad \mathbf{R}^n, \tag{1.2}$$

where $\nu > \frac{n}{n-2}$, $\nu \in \mathbf{R}$.

The corresponding stationary problem of the above equation is as follows:

$$-\Delta w(x) = w(x)|w(x)|^{\nu-1} + f(x) \quad \text{on} \quad \mathbf{R}^n. \tag{1.3}$$

There have been many researches on the Cauchy problem (1.1)–(1.2) without external forces, *i.e.* $f(x) \equiv 0$. Fujita [6] first showed that the Cauchy problem admits a time-global strong solution with $\nu > 1 + 2/n$, provided that $\|a(x)\|_{C^2(\mathbf{R}^n)}$ is sufficiently small. At the same time he also

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