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Absence of resonances for semiclassical Schrödinger operators with Gevrey coefficients

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Abstract. We give lower bounds on resonance free domains for a Schrödinger operator $-h^2\Delta+V$ in the semi-classical limit $h\to 0$, near a non-trapping energy level E_0, when the potential V is dilation analytic at infinity, but only of Gevrey class in a compact set of $\mathbf{R}^n$.

Key words: microlocal spectral asymptotics, semi-classical analysis, h-pseudodifferential operators, Gevrey classes, resonances.

Introduction

In this paper, we give lower bounds on the width of resonance free domains near a non-trapping energy level, for semi-classical operators like

$$P=-h^2\Delta+V(x) \tag{0.1}$$

as V is long range, dilation analytic at infinity but may be only of Gevrey class on a compact set of $\mathbf{R}^n$.

As is well known, (see e.g. [Sj2]), many of the phenomena in semi-classical Quantum Mechanics have their counterpart in geometrical optics. Namely, if one considers the exterior Dirichlet problem for the Helmholtz equation

$$(\Delta+k^2)u=0 \quad \text{in} \quad \mathbf{R}^n\setminus\Omega \ (n \text{ odd}) \tag{0.2}$$

for a bounded domain Ω with smooth boundary, the resonances can be defined as the poles of the scattering matrix in the framework of the Lax-Phillips theory [LaPh], or as the set of $k\in\mathbf{C}$, $\operatorname{Im}k<0$, for which (0.2) has a non-trivial solution in some suitable Hilbert space. If the obstacle is non trapping and has a C^∞ boundary, it follows from the results of propagation of singularities of Melrose-Sjöstrand and Ivrïi (see [Hö Chap. 24]), that there are only finitely many resonances inside any logarithmic neighborhood of

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