

## Finsler metrics of positive constant flag curvature on Sasakian space forms

Aurel BEJANCU and Hani Reda FARRAN

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**Abstract.** Let  $M(c)$  be a Sasakian space form of constant  $\varphi$ -sectional curvature  $c \in (-3, 1)$ . We prove that for any  $K > 0$  there exists a Randers metric on  $M(c)$  of constant flag curvature  $K$ . Moreover, we show that such a Randers metric is not projectively flat. In particular, this means that every odd dimensional sphere admits such metrics.

*Key words:* Finsler manifolds of constant flag curvature, Sasakian space forms, Randers metric.

### 1. Introduction

Finsler metrics of constant flag curvature are unanimously considered to be of great interest in Finsler geometry. Under some growth constraints on the Cartan tensor, Akbar-Zadeh [1] proved that a closed Finsler manifold with constant flag curvature  $K$  is locally Minkowskian if  $K = 0$ , and Riemannian if  $K = -1$ . The case  $K > 0$  is the least understood. Bryant [8] constructed many interesting non-Riemannian examples on the sphere  $S^2$  with  $K = 1$ . Important results on the geometric structure of Finsler manifolds of constant flag curvature  $K = 1$  have been obtained by Shen [11].

Recently, Bao and Shen [5] constructed Randers metrics of constant flag curvature  $K > 1$  on the Lie group  $S^3$ . They have also proved that these Finsler space forms are not projectively flat. We would like to thank professors Bao and Shen for sending this reprint [5] to us. Their results have inspired our work, thus producing the present paper.

Our purpose here is to construct Randers metrics of positive constant flag curvature on a Sasakian space form subject to some constraints on the  $\varphi$ -sectional curvature. More precisely, we prove the following theorem.

**Theorem 1** *Let  $M(c)$  be a Sasakian space form of constant  $\varphi$ -sectional curvature  $c \in (-3, 1)$ . Then for any constant  $K > 0$  there exists a Randers metric  $F$  on  $TM(c)$  such that  $(M(c), F)$  has constant flag curvature  $K$  and*