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On Lagrangian surfaces in $CP^2(\tilde{c})$

(Dedicated to Professor Koichi Ogiue on his sixtieth birthday)

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Abstract. Chen and Ogiue completely classified totally umbilical submanifolds in a non-flat complex-space-form. However, the classification problem of pseudo-umbilical submanifolds in a non-flat complex-space-form is still open. Very recently, Chen introduced the notion of Lagrangian H-umbilical submanifolds which is the simplest totally real submanifolds next to the totally geodesic ones in a complex-space-form and and classified Lagrangian H-umbilical submanifolds in a complex-space-form. The author proved that a Lagrangian H-umbilical surface M in a complex 2-dimensional complex projective space $CP^2(\tilde{c})$ is an isotropic surface in $CP^2(\tilde{c})$ if and only if M is a minimal surface in $CP^2(\tilde{c})$. In this paper, firstly, we prove that a Lagrangian surface M in $CP^2(\tilde{c})$ is an isotropic surface in $CP^2(\tilde{c})$ if and only if M is a minimal surface in $CP^2(\tilde{c})$. Secondly, we classify Lagrangian non-totally geodesic pseudo-umbilical surfaces in $CP^2(\tilde{c})$.

Key words: totally real, isotropic, pseudo-umbilical, Lagrangian H-umbilical.

1. Introduction

Let M be an n-dimensional submanifold of a complex m-dimensional Kaehler manifold $\tilde{M}$ with complex structure J and Kaehler metric g. A submanifold M of a Kaehler manifold $\tilde{M}$ is said to be a *totally real* if each tangent space of M is mapped into the normal space by the complex structure of $\tilde{M}$ (see Chen and Ogiue [5]). The totally real submanifold M of $\tilde{M}$ is called *Lagrangian* if $n = m$. A Kaehler manifold of constant holomorphic sectional curvature $\tilde{c}$ is called a *complex-space-form* and will be denoted by $\tilde{M}(\tilde{c})$. Let $CP^m(\tilde{c})$ be a complex m-dimensional complex projective space with the Fubini-Study metric of constant holomorphic sectional curvature $\tilde{c}$.

Chen and Ogiue [6] classified totally umbilical submanifolds in a non-flat complex-space-form $\tilde{M}^m(\tilde{c})$ $(\tilde{c} \neq 0)$ and proved that a non-flat complex-space-form $\tilde{M}^m(\tilde{c})$ $(\tilde{c} \neq 0)$ $(m \geq 2)$ admits no totally umbilical, Lagrangian submanifolds except the totally geodesic ones.

Very recently, Chen [1] introduced the notion of Lagrangian H-umbilical submanifolds which is the simplest totally real submanifolds next to

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