The joint approximate point spectrum of an operator

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Abstract. A new proof of a result, due to Xia, concerning the joint approximate point spectrum of an operator is given. This result is then applied to obtain certain spectral properties for operators, such as p-hyponormal and log-hyponormal operators, which have the identical approximate and joint approximate point spectra.

Key words: approximate and joint approximate point spectra, p- and log-hyponormal operators, invariant subspace.

1. Introduction

Let T be a bounded linear operator on a Hilbert space H. A complex number $\lambda \in \mathbb{C}$ is said to be in the approximate point spectrum $\sigma_a(T)$ of the operator T if there is a sequence $\{x_n\}$ of unit vectors satisfying $(T - \lambda)x_n \to 0$. If in addition, $(T^* - \overline{\lambda})x_n \to 0$, then λ is said to be in the joint approximate point spectrum $\sigma_{ja}(T)$ of T. The boundary $\partial\sigma(T)$ of the spectrum $\sigma(T)$ of the operator T is known to be a subset of $\sigma_a(T)$. Although, in general, one has $\sigma_{ja}(T) \subset \sigma_a(T)$, there are many classes of operators T for which

$$\sigma_{ja}(T) = \sigma_a(T). \tag{1}$$

For example, if T is either normal or hyponormal, then T satisfies (1). More generally, (1) holds if T is semi-hyponormal [15], p-hyponormal [7] or log-hyponormal [14], [4, Corollary 4.5]. In [10], Duggal introduced a class $K(p)$ of operators which contains the p-hyponormal operators and showed [10, Theorem 4] that operators T in the class $K(p)$ satisfy (1).

In this paper we give a new proof of a result, due to Xia [15], concerning the joint approximate point spectrum of an operator. The result is then applied to obtain certain spectral properties for operators T for which (1) is satisfied. All operators considered in this paper are assumed to be bounded linear operators on the Hilbert space H. This paper may be considered, for