Hokkaido Mathematical Journal Vol. 31 (2002) p. 137–149

Smooth unique solutions for a modified Mullins-Sekerka model arising in diblock copolymer melts

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(Received June 12, 2000; Revised November 15, 2000)

Abstract. Of concern is a modified Mullins-Sekerka model arising in diblock copolymer melts. As the new feature of this system a nonlocal inhomogeneous term is introduced. It is shown that the corresponding moving boundary problem is classically well posed.

Key words: Mullins-Sekerka flow, Hele-Shaw flow, Cahn-Hilliard equation, free boundary problem, diblock copolymer melt, convexity, curvature.

1. Introduction

In [18] a modified Cahn-Hilliard equation is proposed to study micro-phase separation of diblock copolymer. Let Ω be a bounded domain in $\mathbb{R}^n$ with a smooth boundary $\partial\Omega$ and consider the following parabolic initial boundary value problem

$$\left\{\begin{array}{rll} u_t+\Delta(\varepsilon^2\Delta u+W'(u))-\sigma(u-\overline{u}_0)=0 & \text{in} & \Omega\times(0,\infty) \\ \partial_\nu u=\partial_\nu\Delta u=0 & \text{on} & \partial\Omega\times[0,\infty) \\ u(0,\cdot)=u_0 & \text{in} & \Omega, \end{array}\right. \tag{1.1}$$

where ε and σ are positive contants and W stands for a double-well potential with global minima at ± 1. Moreover, $\overline{u}_0:=\frac{1}{|\Omega|}\int_\Omega u_0\,dx$, with $|\Omega|$ being the Lebesgue measure of Ω, and $\partial_\nu u$ stands for the derivative of u with respect to the outer unit normal ν on $\partial\Omega$. In the case $\sigma=0$ system (1.1) reduces to the usual Cahn-Hilliard model, cf. [21]. However, if one considers separation of diblock copolymer, the effect of nonlocality should be taken into account, which stems from a long-range interaction of diblock copolymer. The third term of the left-hand side of the first equation above comes from the nonlocal term associated to Gibbs energy and the parameter σ is inversely proportional to the square of the total chain length of the

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2000 Mathematics Subject Classification : 35R35, 35J05, 35B50, 53A07.

The first author is grateful to Prof. Y. Giga and to Dr. K. Ito for their kind hospitality and for the stimulating discussions during his stay at Hokkaido University.